Student Workbook

Unit Review — Trig Identities & Equations

Name: ______________________ Date: ______________ Class: ________

UNIT SUMMARY

Consolidate your understanding of simplifying and proving identities, sum and difference formulas, double-angle and half-angle formulas, product-to-sum conversions, solving trigonometric equations, and modeling periodic phenomena.

Formula Reference

$$\sin^2\theta + \cos^2\theta = 1 \mid 1 + \tan^2\theta = \sec^2\theta \mid 1 + \cot^2\theta = \csc^2\theta$$

Sum/Diff	$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$; $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$
2θ Forms	$\sin(2\theta)=2\sin\theta\cos\theta$; $\cos(2\theta)=\cos^2\theta-\sin^2\theta=2\cos^2\theta-1=1-2\sin^2\theta$
Half-Angle	$\sin(\theta/2)=\pm\sqrt{(1-\cos\theta)/2}$; $\cos(\theta/2)=\pm\sqrt{(1+\cos\theta)/2}$
Prod-Sum	$\sin\alpha\cos\beta=½[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$; $\sin\alpha+\sin\beta=2\sin((\alpha+\beta)/2)\cos((\alpha-\beta)/2)$
Modeling	$y=A\cdot\sin(Bx+C)+D$; $A=(\max-\min)/2$; $D=(\max+\min)/2$; $B=2\pi/\text{period}$; phase shift$=-C/B$

Worked Examples

EXAMPLE 1

Prove: $(1 + \sin\theta)/\cos\theta + \cos\theta/(1 + \sin\theta) = 2\sec\theta$

- Combine over common denominator $(1+\sin\theta)\cos\theta$: $[(1+\sin\theta)^2 + \cos^2\theta] / [(1+\sin\theta)\cos\theta]$
- Expand numerator: $1+2\sin\theta+\sin^2\theta+\cos^2\theta = 1+2\sin\theta+1 = 2+2\sin\theta = 2(1+\sin\theta)$
- Divide: $2(1+\sin\theta) / [(1+\sin\theta)\cos\theta] = 2/\cos\theta = 2\sec\theta$ ✓

Answer: Identity proved: $(1+\sin\theta)/\cos\theta + \cos\theta/(1+\sin\theta) = 2\sec\theta$

EXAMPLE 2

Find the exact value of sin(22.5°) using a half-angle formula

- $22.5° = 45°/2$, so use $\sin(\theta/2)$ with $\theta=45°$
- $\sin(22.5°) = \sqrt{(1-\cos45°)/2} = \sqrt{(1-\sqrt{2}/2)/2} = \sqrt{(2-\sqrt{2})/4} = \sqrt{2-\sqrt{2}}/2$
- 22.5° is in QI, so the sign is positive

Answer: $\sin(22.5°) = \sqrt{2 - \sqrt{2}}/2$

EXAMPLE 3

Solve: $2\cos^2\theta - 3\cos\theta + 1 = 0$ on $[0, 2\pi)$

- Factor: $(2\cos\theta - 1)(\cos\theta - 1) = 0$
- $\cos\theta = 1/2$: $\theta = \pi/3, 5\pi/3$
- $\cos\theta = 1$: $\theta = 0$

Answer: $\theta = 0, \pi/3, 5\pi/3$

EXAMPLE 4

Write sin(4x)cos(2x) as a sum

- Use $\sin\alpha \cos\beta = ½[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$ with $\alpha=4x$, $\beta=2x$
- $\sin(4x)\cos(2x) = ½[\sin(6x) + \sin(2x)]$

Answer: $\sin(4x)\cos(2x) = ½[\sin(6x) + \sin(2x)]$

EXAMPLE 5

A spring's displacement is $d(t) = 6\cos(2\pi t/3) - 2$. Find the first time $t > 0$ when $d(t) = 1$

- Set $6\cos(2\pi t/3) - 2 = 1 \rightarrow 6\cos(2\pi t/3) = 3 \rightarrow \cos(2\pi t/3) = 1/2$
- Reference angle: $\pi/3$. Cosine is positive in QI and QIV
- $2\pi t/3 = \pi/3 \rightarrow t = 1/2$. Or $2\pi t/3 = 5\pi/3 \rightarrow t = 5/2$

Answer: First time: $t = 1/2$ second

Common Mistakes to Avoid

✗ Moving terms across the equals sign when proving an identity.	✓ Work on one side only. Transform it using known identities until it matches the other side.
✗ Writing $\cos(\alpha+\beta) = \cos\alpha + \cos\beta$.	✓ $\cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$. Trig functions do not distribute over addition.
✗ Using the wrong form of $\cos(2\theta)$ for the given information.	✓ Choose the form that uses what you know: $1-2\sin^2\theta$ if you know $\sin\theta$; $2\cos^2\theta-1$ if you know $\cos\theta$.
✗ Determining the ± sign in a half-angle formula from the quadrant of θ.	✓ The sign is determined by the quadrant of $\theta/2$, not θ.

✗ Finding only one solution to a trig equation and forgetting the second quadrant.

✓ Every equation $\sin\theta=k$ or $\cos\theta=k$ with $|k|<1$ has two solutions per period. Always check both quadrants.

✗ Setting B equal to the period in a sinusoidal model.

✓ $B = 2\pi/\text{period}$. If the period is 12, then $B = \pi/6$.

Math Tips

★ When stuck proving an identity, convert everything to $\sin\theta$ and $\cos\theta$ — it almost always reveals the path.

★ The three Pythagorean identities all come from $\sin^2\theta+\cos^2\theta=1$. Divide by $\cos^2\theta$ to get $1+\tan^2\theta=\sec^2\theta$; divide by $\sin^2\theta$ to get $\cot^2\theta+1=\csc^2\theta$.

★ For half-angle problems, always determine the quadrant of $\theta/2$ before choosing the sign.

★ When solving trig equations with double angles, double the domain first, find all solutions, then halve them.

★ The four-step recipe for sinusoidal models: (1) $A=(\max-\min)/2$, (2) $D=(\max+\min)/2$, (3) $B=2\pi/\text{period}$, (4) find C from a known point.

★ Product-to-sum formulas are the key to integrating products of trig functions in calculus — they convert a product into a sum that can be integrated term by term.